9-4 Common Logarithms

Main Ideas

- Solve exponential equations and inequalities using common logarithms.
- Evaluate logarithmic expressions using the Change of Base Formula.

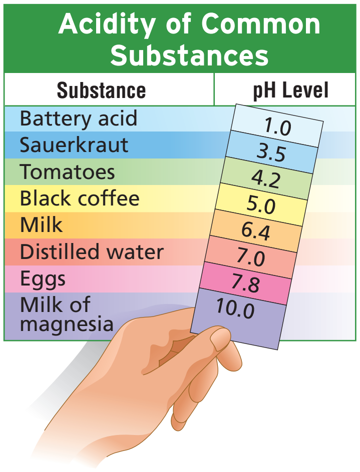
New Vocabulary

common logarithm
Change of Base Formula

GET READY for the Lesson

The pH level of a substance measures its acidity. A low pH indicates an acid solution while a high pH indicates a basic solution. The pH levels of some common substances are shown.

The pH level of a substance is given by pH $= -\log_{10} [H^+]$, where H^+ is the substance's hydrogen ion concentration in moles per liter. Another way of writing this formula is pH $= -\log [H^+]$.



Acidity of Common Substances

Substance	pH Level
Battery acid	1.0
Sauerkraut	3.5
Tomatoes	4.2
Black coffee	5.0
Milk	6.4
Distilled water	7.0
Eggs	7.8
Milk of magnesia	10.0

Common Logarithms You have seen that the base 10 logarithm function, $y = \log_{10} x$, is used in many applications. Base 10 logarithms are called **common logarithms**. Common logarithms are usually written without the subscript 10.

$$\log_{10} x = \log x,\ x > 0$$

Most scientific calculators have a $\boxed{\text{LOG}}$ key for evaluating common logarithms.

EXAMPLE Find Common Logarithms

① **Use a calculator to evaluate each expression to four decimal places.**

a. **log 3**

KEYSTROKES: $\boxed{\text{LOG}}$ 3 $\boxed{\text{ENTER}}$ .4771212547

log 3 is about 0.4771.

b. **log 0.2**

KEYSTROKES: $\boxed{\text{LOG}}$ 0.2 $\boxed{\text{ENTER}}$ –.6989700043

log 0.2 is about –0.6990.

CHECK Your Progress

1A. log 5

1B. log 0.5

Study Tip

Technology

Nongraphing scientific calculators often require entering the number followed by the function, for example, 3 $\boxed{\text{LOG}}$.

Sometimes an application of logarithms requires that you use the inverse of logarithms, or exponentiation.

$$10^{\log x} = x$$

Real-World EXAMPLE Solve Logarithmic Equations

2 **EARTHQUAKES** **The amount of energy E, in ergs, that an earthquake releases is related to its Richter scale magnitude M by the equation $\log E = 11.8 + 1.5M$. The Chilean earthquake of 1960 measured 8.5 on the Richter scale. How much energy was released?**

$\log E = 11.8 + 1.5M$	Write the formula.
$\log E = 11.8 + 1.5(8.5)$	Replace M with 8.5.
$\log E = 24.55$	Simplify.
$10^{\log E} = 10^{24.55}$	Write each side using exponents and base 10.
$E = 10^{24.55}$	Inverse Property of Exponents and Logarithms
$E \approx 3.55 \times 10^{24}$	Use a calculator.

The amount of energy released by this earthquake was about 3.55×10^{24} ergs.

CHECK Your Progress

2. Use the equation above to find the energy released by the 2004 Sumatran earthquake, which measured 9.0 on the Richter scale and led to a tsunami.

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If both sides of an exponential equation cannot easily be written as powers of the same base, you can solve by taking the logarithm of each side.

EXAMPLE Solve Exponential Equations Using Logarithms

3 **Solve $3^x = 11$.**

$3^x = 11$	Original equation
$\log 3^x = \log 11$	Property of Equality for Logarithmic Functions
$x \log 3 = \log 11$	Power Property of Logarithms
$x = \dfrac{\log 11}{\log 3}$	Divide each side by log 3.
$x \approx \dfrac{1.0414}{0.4771}$	Use a calculator.
$x \approx 2.1828$	

The solution is approximately 2.1828.

CHECK You can check this answer using a calculator or by using estimation. Since $3^2 = 9$ and $3^3 = 27$, the value of x is between 2 and 3. In addition, the value of x should be closer to 2 than 3, since 11 is closer to 9 than 27. Thus, 2.1828 is a reasonable solution. ✓

Study Tip

Using Logarithms

When you use the Property of Equality for Logarithmic Functions, this is sometimes referred to as *taking the logarithm of each side.*

CHECK Your Progress

Solve each equation.

3A. $4^x = 15$

3B. $6^x = 42$

EXAMPLE Solve Exponential Inequalities Using Logarithms

4 **Solve $5^{3y} < 8^{y-1}$.**

$5^{3y} < 8^{y-1}$	Original inequality
$\log 5^{3y} < \log 8^{y-1}$	Property of Inequality for Logarithmic Functions
$3y \log 5 < (y - 1) \log 8$	Power Property of Logarithms
$3y \log 5 < y \log 8 - \log 8$	Distributive Property
$3y \log 5 - y \log 8 < -\log 8$	Subtract y log 8 from each side.
$y(3 \log 5 - \log 8) < -\log 8$	Distributive Property
$y < \dfrac{-\log 8}{3 \log 5 - \log 8}$	Divide each side by 3 log 5 − log 8.
$y < -0.7564$	Use a calculator.

The solution set is $\{y \mid y < -0.7564\}$.

CHECK Test $y = -1$.

$5^{3y} < 8^{y-1}$	Original inequality
$5^{3(-1)} < 8^{(-1)-1}$	Replace y with −1.
$5^{-3} < 8^{-2}$	Simplify.
$\dfrac{1}{125} < \dfrac{1}{64}$ ✓	Negative Exponent Property

Study Tip

Solving Inequalities

Remember that the direction of an inequality must be switched if both sides are multiplied or divided by a negative number. Since $3 \log 5 - \log 8 > 0$, the inequality does not change.

CHECK Your Progress **Solve each inequality.**

4A. $3^{2x} \geq 6^{x+1}$

4B. $4^{y} < 5^{2y+1}$

Change of Base Formula The **Change of Base Formula** allows you to write equivalent logarithmic expressions that have different bases.

KEY CONCEPT *Change of Base Formula*

Symbols For all positive numbers, a, b and n, where $a \neq 1$ and $b \neq 1$,

$$\log_a n = \frac{\log_b n}{\log_b a}. \quad \begin{matrix} \leftarrow \text{log base } b \text{ of original number} \\ \leftarrow \text{log base } b \text{ of old base} \end{matrix}$$

Example $\log_5 12 = \dfrac{\log_{10} 12}{\log_{10} 5}$

To prove this formula, let $\log_a n = x$.

$a^x = n$	Definition of logarithm
$\log_b a^x = \log_b n$	Property of Equality for Logarithms
$x \log_b a = \log_b n$	Power Property of Logarithms
$x = \dfrac{\log_b n}{\log_b a}$	Divide each side by $\log_b a$.
$\log_a n = \dfrac{\log_b n}{\log_b a}$	Replace x with $\log_a n$.

The Change of Base Formula makes it possible to evaluate a logarithmic expression of any base by translating the expression into one that involves common logarithms.

EXAMPLE Change of Base Formula

5 **Express $\log_4 25$ in terms of common logarithms. Then approximate its value to four decimal places.**

$$\log_4 25 = \frac{\log_{10} 25}{\log_{10} 4} \quad \text{Change of Base Formula}$$

$$\approx 2.3219 \quad \text{Use a calculator.}$$

The value of $\log_4 25$ is approximately 2.3219.

CHECK Your Progress

5. Express $\log_6 8$ in terms of common logarithms. Then approximate its value to four decimal places.

CHECK Your Understanding

Example 1 (p. 528)

Use a calculator to evaluate each expression to four decimal places.

1. $\log 4$ **2.** $\log 23$ **3.** $\log 0.5$

Example 2 (p. 529)

4. **NUTRITION** For health reasons, Sandra's doctor has told her to avoid foods that have a pH of less than 4.5. What is the hydrogen ion concentration of foods Sandra is allowed to eat? Use the information at the beginning of the lesson.

Example 3 (p. 529)

Solve each equation. Round to four decimal places.

5. $9^x = 45$ **6.** $3.1^{a-3} = 9.42$

7. $11^{x^2} = 25.4$ **8.** $7^{t-2} = 5^t$

Example 4 (p. 530)

Solve each inequality. Round to four decimal places.

9. $4^{5n} > 30$ **10.** $4^{p-1} \leq 3^p$

Example 5 (p. 531)

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

11. $\log_7 5$ **12.** $\log_3 42$ **13.** $\log_2 9$

Exercises

HOMEWORK HELP

For Exercises	See Examples
14–19	1
20, 21	2
22–27	3
28–33	4
34–39	5

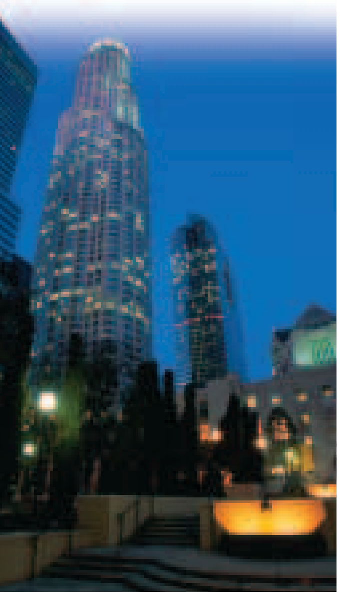
Use a calculator to evaluate each expression to four decimal places.

14. $\log 5$ **15.** $\log 12$ **16.** $\log 7.2$

17. $\log 2.3$ **18.** $\log 0.8$ **19.** $\log 0.03$

20. **POLLUTION** The acidity of water determines the toxic effects of runoff into streams from industrial or agricultural areas. A pH range of 6.0 to 9.0 appears to provide protection for freshwater fish. What is this range in terms of the water's hydrogen ion concentration?

21. **BUILDING DESIGN** The 1971 Sylmar earthquake in Los Angeles had a Richter scale magnitude of 6.3. Suppose an architect has designed a building strong enough to withstand an earthquake 50 times as intense as the Sylmar quake. Find the magnitude of the strongest quake this building can withstand.



Real-World Link

There are an estimated 500,000 detectable earthquakes in the world each year. Of these earthquakes, 100,000 can be felt and 100 cause damage.

Source: earthquake.usgs.gov

Solve each equation or inequality. Round to four decimal places.

22. $5^x = 52$ **23.** $4^{3p} = 10$ **24.** $3^{n+2} = 14.5$

25. $9^{z-4} = 6.28$ **26.** $8.2^{n-3} = 42.5$ **27.** $2.1^{t-5} = 9.32$

28. $6^x \geq 42$ **29.** $8^{2a} < 124$ **30.** $4^{3x} \leq 72$

31. $8^{2n} > 52^{4n+3}$ **32.** $7^{p+2} \leq 13^{5-p}$ **33.** $3^{y+2} \geq 8^{3y}$

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

34. $\log_2 13$ **35.** $\log_5 20$ **36.** $\log_7 3$

37. $\log_3 8$ **38.** $\log_4 (1.6)^2$ **39.** $\log_6 \sqrt{5}$

ACIDITY **For Exercises 40–43, use the information at the beginning of the lesson to find each pH given the concentration of hydrogen ions.**

40. ammonia: $[H^+] = 1 \times 10^{-11}$ mole per liter

41. vinegar: $[H^+] = 6.3 \times 10^{-3}$ mole per liter

42. lemon juice: $[H^+] = 7.9 \times 10^{-3}$ mole per liter

43. orange juice: $[H^+] = 3.16 \times 10^{-4}$ mole per liter

Solve each equation. Round to four decimal places.

44. $20^{x^2} = 70$ **45.** $2^{x^2-3} = 15$ **46.** $2^{2x+3} = 3^{3x}$

47. $16^{d-4} = 3^{3-d}$ **48.** $5^{5y-2} = 2^{2y+1}$ **49.** $8^{2x-5} = 5^{x+1}$

50. $2^n = \sqrt{3^{n-2}}$ **51.** $4^x = \sqrt{5^{x+2}}$ **52.** $3^y = \sqrt{2^{y-1}}$

MUSIC **For Exercises 53 and 54, use the following information.**
The musical cent is a unit in a logarithmic scale of relative pitch or intervals. One octave is equal to 1200 cents. The formula to determine the difference in cents between two notes with frequencies a and b is $n = 1200\left(\log_2 \frac{a}{b}\right)$.

53. Find the interval in cents when the frequency changes from 443 Hertz (Hz) to 415 Hz.

54. If the interval is 55 cents and the beginning frequency is 225 Hz, find the final frequency.

MONEY **For Exercises 55 and 56, use the following information.**
If you deposit P dollars into a bank account paying an annual interest rate r (expressed as a decimal), with n interest payments each year, the amount A you would have after t years is $A = P\left(1 + \frac{r}{n}\right)^{nt}$. Marta places $100 in a savings account earning 2% annual interest, compounded quarterly.

55. If Marta adds no more money to the account, how long will it take the money in the account to reach $125?

56. How long will it take for Marta's money to double?

EXTRA PRACTICE
See pages 910, 934.

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H.O.T. Problems

57. **CHALLENGE** Solve $\log_{\sqrt{a}} 3 = \log_a x$ for x and explain each step.

58. Write $\dfrac{\log_5 9}{\log_5 3}$ as a single logarithm.

59. **CHALLENGE**

- **a.** Find the values of $\log_2 8$ and $\log_8 2$.
- **b.** Find the values of $\log_9 27$ and $\log_{27} 9$.
- **c.** Make and prove a conjecture about the relationship between $\log_a b$ and $\log_b a$.

60. ***Writing in Math*** Use the information about acidity of common substances on page 528 to explain why a logarithmic scale is used to measure acidity. Include the hydrogen ion concentration of three substances listed in the table, and an explanation as to why it is important to be able to distinguish between a hydrogen ion concentration of 0.00001 mole per liter and 0.0001 mole per liter in your answer.

STANDARDIZED TEST PRACTICE

61. **ACT/SAT** If $2^4 = 3^x$, then what is the approximate value of x?

A 0.63

B 2.34

C 2.52

D 4

62. **REVIEW** Which equation is equivalent to $\log_4 \frac{1}{16} = x$?

F $\frac{1^4}{16} = x^4$

G $\left(\frac{1}{16}\right)^4 = x$

H $4^x = \frac{1}{16}$

J $4^{\frac{1}{16}} = x$

Spiral Review

Use $\log_7 2 \approx 0.3562$ and $\log_7 3 \approx 0.5646$ to approximate the value of each expression. (Lesson 9-3)

63. $\log_7 16$

64. $\log_7 27$

65. $\log_7 36$

Solve each equation or inequality. Check your solutions. (Lesson 9-2)

66. $\log_4 r = 3$

67. $\log_8 z \leq -2$

68. $\log_3 (4x - 5) = 5$

69. Use synthetic substitution to find $f(-2)$ for $f(x) = x^3 + 6x - 2$. (Lesson 6-7)

70. **MONEY** Viviana has two dollars worth of nickels, dimes, and quarters. She has 18 total coins, and the number of nickels equals 25 minus twice the number of dimes. How many nickels, dimes, and quarters does she have? (Lesson 3-5)

GET READY for the Next Lesson

PREREQUISITE SKILL **Write an equivalent exponential equation.** (Lesson 9-2)

71. $\log_2 3 = x$

72. $\log_3 x = 2$

73. $\log_5 125 = 3$